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Qualification Routes Messaging for Dynamic Systems Using Logical-Probabilistic Method

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The problem of message routes evaluation in a dynamic network of mobile subscribers is investigated. The network of mobile objects is represented by a graph with varying in time structure. The searching of the optimal route at any given time has polynomial complexity. As a solution, logical–probabilistic method to build the estimates of routes has been proposed. This method allows to obtain an analytical expression of the message delivery probability function for s–connected graph with a given dimension. In this case, the time of searching the optimal route can be considered as a constant.

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1. Introduction

At the present time, more and more attention is paid to research problems of building cloud–oriented mesh–network of vehicles. Continuous improvement of data transmission assets, network equipment, internet working methods and cloud services access methods allow to define new tasks of provisioning the information services to mobile networks subscribers. Special interest is presented by the problems of improving the quality of messaging between traffic networks participants in areas with low–quality network coverage [1].

Messaging between a vehicle and cloud environment is supplied via a dedicated channel, which is organized with the support of telematics hardware equipment [2]. Quality of service in cloud–oriented environment is determined by the reliability of third party hardware equipment and the size of the coverage area of the cellular networks.

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As a perspective trend in development of information network in areas with unstable signal reception is usually considered as a mobile self–organizing local network of vehicles with access points to cloud environment. In this model the exchange of messages between a vehicle and a cloud can be routed by different paths, then the set of vehicles on the road is presented by wireless LAN with varying topology, with a variable number of points which can communicate with the cloud environment. Currently, wireless messaging methods are supported by various techniques such as: Wi–Fi (802.11bg), mesh (802.11s), DSRC, LTE(4G), UMTS(3G), which are implemented in vehicle's on-board hardware equipment [3].

The probability of delivering messages using these technologies may vary significantly at any given moment depending on repeaters location, terrain properties, intensity of data exchange and the amount of data.

Thus, problem of choosing an optimal route between two messaging nodes at time T_i arises.

Mobile objects connected each other at time T_i and used the same connection technology are build the dynamic network. The network configuration and reliability changes over time.

It is necessary to develop a formal criterion for the well–grounded choice of the best data path at T_i. In this paper, the probability of message delivery is considered as such a criterion.

2. Problem analysis

A dynamic network of mobile objects can be represented as a graph $G(U, L)$ where U_i is a mobile object, L_{ij} is a communication channels between mobile objects. Each data transmission technology determines a subnet, i.e. G_m is the subgraph $G_m \subset G(U, L)$. Set $F(G_m)$ as a function of the probability of the message delivery, defined on the subgraph G_m. T_i is the time moment of the message transmission. In this case, statement of the problem will be as follows: in time moment T_i determine $\max\{F_{(G_m)}\}, m = 1, \ldots, n$.

Feature of this formulation of the problem is the restriction on the computation time F, as the dynamics of changing parameters G_m is high.

Similar problems have been solved in the works of Floyd, Dijkstra, Levit, which represent fundamental research methods for building routes on graphs. In the work it was presented Dijkstra algorithm [4] for finding the shortest distance from one of the nodes to each other, the complexity of the algorithm in the worst case is $O(n^2)$. Floyd–Warshall algorithm [5] has the complexity $O(n^3)$, and the Levit's algorithm [5], which is a modified version of the Bellman–Ford's algorithm, in worst case has exponential complexity. However, in practice, the algorithms show good results, with logarithmic complexity $O(M \log N)$ where N is the set of nodes and M — set of edges between nodes. These approaches show good results for routing problems on networks with low dynamics changes connected nodes, examples of software implementation of such algorithms are dynamic routing protocols: OSPF and IS–IS. At the same time, their implementation in our task does not provide a solution for a linear or constant time. Algorithms for finding the shortest path on the graph belong to the class P, and relate to problems with polynomial complexity. This complexity is due to the necessity to recalculate all routes in the graph, which structure changes in time moment T_i.

Current state of problem solution: the logical-probabilistic calculus allows to connect a Boolean algebra [6] with operation of the circuit, and network data transmission systems. This allows to move away from the use of classical routing algorithms on graphs and move to recalculate notation logical variables and probability functions that allow quick evaluation form to submit transmission in constant time in a prearranged structure [7].

3. Assessment of the reliability of message delivery by using logical–probabilistic method

Definition of logical–probabilistic method is to use mixed form of the probability function (MFPF) [8], describing in a compact form the set of conditional probabilities. It takes into account the conditions of occurrence of an event in the form of logical variables and functions in terms of the degree of probability of events.

Logical–probabilistic method (LPM) originally designed to work with static structures where communication between nodes in the graph is given in terms of the problem, in this structure only communication channels state changes. In order to move to a dynamic system and to ensure constant time route calculation it is proposed to introduce dummy nodes, which will manage the appearance and disappearance of vehicles (nodes) at a particular time T_i. Then a calculation formula LPM could be drawn for the case of the maximum number of nodes (upper limit), and the performance of channels in fictitious nodes before the actual occurrence of the car set to false.

Consider the application of logical–probabilistic method, to build estimates of the probability of message delivery from vehicle A_n to A_1 at a specified time moment T_i into network, with incomplete notification node about network status [9].

Graph structure is assumed as follows: vehicle network is represented as a directed acyclic graph. In this graph, each node represents a switching center and receiving messages, i.e. each message can be transmitted into n-th destination, if it exists, or a direct path to the destination, or by constructing a chain of hops on workable channels. An example of such a graph is shown in Fig. 1.

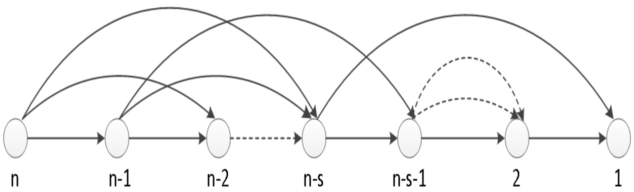

FIG. 1: s–connected oriented structure.

Condition of the working system is defined as follows: the system works, if there is at least one workable route from node n to 1. In turn, the route is operational (workable) if all its nodes and links between nodes are in the work state.

Introduce the following notation:
R_c — working node probability,
n — number of nodes in graph (network),
s — number of link between nodes in graph (network),
f_k — logical function of successful connection between nodes,
$k_{i_{k_s}}$ — number of priority link, which is given by the coefficient of availability k_s-th link,
P_k — total probability availability k-th link,
x_k — logical variable of availability k-th node,
$x_{k,\ k-1}$ — logical variable of availability link between $k-1$ and k-th nodes,
Q_i — probability of failure i–th link.

For every node, let us define the priority links: ki_{k1}, ki_{k2}, $\ldots$, ki_{ks} for $k > s$ and ki_{k1}, ki_{k2}, $\ldots$, $ki_{k,k-1}$ for $k \leqslant s$. Assume for definiteness, transmission is over the communication channel is working properly in the direction of the lowest numbered node. Then the priority links for all routes from node k to 1 look like $(k, k-s), (k, k-s+1), \ldots, (k, k-1)$ when $k > s$ and $k1, k2, \ldots, (k, k-1)$ when $(k \leqslant s)$.

Condition of network with incomplete input data about elements corresponds to uncertainty of the status of all nodes and links in route, by which the message is transmitted. In case of failure of one or more elements of the route the message will be lost.

Logic–probabilistic method for estimating the probability of the message delivery from node k to node 1, consists of two stages:

1. Building logical function of successful connection between nodes k and 1.

2. Building total probability formula of successful connection between nodes k and 1.

The second stage is realized by substitution method of logical variables and MFPF building.

Look at the first stage [8].

Find the calculation formula for the probability R_c of message delivery from k–th node. Define f_k as logical function of successful connection between nodes k and 1. Then f_k is realized in the form of the recurrence relation:

$$f_k = x_k(x_{k,k-s}f_{k-s} \vee x'_{k,k-s}(x_{k,k-s+1}f_{k-s+1} \vee \ \ldots \ \vee x'_{k,k-s}(x_{k,k-2}f_{k-2} \vee x'_{k,k-2}x_{k,k-1}f_{k-1}) \ldots)),$$
$$s+1 \leqslant k \leqslant n, n > s \qquad (1)$$

where n is a number of nodes in graph, x_k is a logical variable of availability k-th node, x'_k is a logical variable shows inoperable state of k-th node. Logical function f_k reflects all the many routes to deliver message in s-connected oriented graph. For example, the expression $x_k x_{k,k-s} f_{k-s}$ is interpreted as sending a message through a node using route $x_{k,k-s}$ and recurrence relation logical function with value f_{k-s}. If route $x'_{k,k-s}$ fails, then the next route — $x_{k,k-s+1}$ will be used and recursive function is calculated for the next value — f_{k-s+1}. In case of $2 \leqslant k \leqslant s$, the function will be as follows:

$$f_k = x_k(x_{k1}f_1 \vee x'_{k1}(x_{k2}f_2 \vee \dots \vee x'_{k,k-3}(x_{k,k-2}f_{k-2} \vee x'_{k,k-2}x_{k,k-1}f_{k-1})\dots)),\ 2 \leqslant k \leqslant s. \qquad (2)$$

Look at the second stage.

By the orthogonality of the terms in 1 and 2, the substitution variables can be carried out in each term separately. MFPF takes the following form:

$$P_k^{(k-1)}(f_{k-1},\dots,f_{k-s}) = R_k\Big(1 - Q_{k,k-s}^{f_{k-s}} + \sum_{i=k-s+1}^{k-1}\prod_{j=1}^{i-k+s} Q_{k,i-j}\big(1-Q_{ki}^{f_i}\big)\Big),$$
$$s+1 \leqslant k \leqslant n-1;$$
$$P_k^{(k-1)}(f_{k-1},\dots,f_1) = R_k\Big(1 - Q_{k1}^{f_1} + \sum_{i=2}^{k-1}\prod_{j=1}^{i-1} Q_{k,\ i-j}\big(1-Q_{ki}^{f_i}\big)\Big), 2 \leqslant k \leqslant s \qquad (3)$$

where P is an estimated value for total probability message delivery from n-th node to 1, Q_k is a probability of node failure k-th node, R_k is a probability of availability k-th node. $Q_{k,\ k-s}$ is a probability of route failure between k-th and $(k-s)$-th nodes.

By analogy, the substitution is carried out for each term of the expression 3. It is convenient to start substitution in functions with small number of k.

After all substitution, we obtain an expression for the total probability of message delivery from k–th node to 1. According to 3 for arbitrary numbers $k \leqslant s$ we obtain:

$$P_k = R_k(R_{k1}P_1 + Q_{k1}(P_2R_{k2} + Q_{k2}(R_{k3}P_3 + \dots + Q_{k,k-2}R_{k,k-1}P_{k-1})). \qquad (4)$$

Similarly for $k > s$:

$$P_k = R_k(R_{k,k-s}P_{k-s} + Q_{k,k-s} \times(R_{k,k-s+1}P_{k-s+1} + \dots + Q_{k,k-2}R_{k,k-1}P_{k-1})). \qquad (5)$$

Thus, knowing the vector of probabilities of availability R_i and R_{ij} we can calculate the probability of message delivery from k-th node to 1 for each channel technology. For m available channels by different technologies it is easy to get m values of probability message delivery, and the computational complexity of the calculation will be no higher than the linear.

4. Example of calculating the probability of message delivery

Consider an example of calculating the probability of message delivery between k–th nodes with two different channel technologies a_i b_i $i = 1\dots k$ on each node. Transmitters of each technology are combined in isolated networks A and B. In this case, the function for calculating the probability is used as a decision rule in the route selection process for transmitting messages between nodes through a network A or B. Let each network be defined by the fully connected topology, which is presented in Fig. 2.

Calculation example for the first four nodes is shown below in the formulas (6, 7). Calculation of the elements to the k-th element is carried out according to the MFPF formula (4) recurrently substituting the corresponding $k-1$ states.

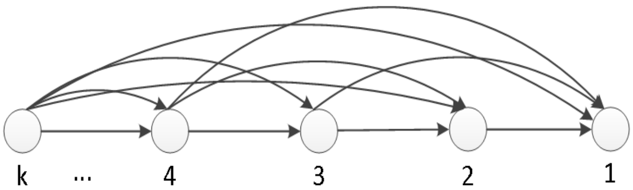

FIG. 2. Network topology example for different channel technologies.

This formula evaluation is based on formulas (1) and (2). Then logic function of success communication nodes for each of the two channels becomes:

$$\begin{aligned} f_4(P_4=1) &= x_4(x_{34}x_3 \vee x_{24}f_2 \vee x_{41}x_1), \\ f_4 &= x_1(x_{34}f_3 \vee x_{24}f_2 \vee x_{14}f_1), \\ f_3 &= x_3(x_{23}f_2 \vee x_{13}f_1), \\ f_2 &= x_2(x_{12}f_1), \\ f_1 &= x_1. \end{aligned} \quad (6)$$

For example, for the first four nodes MFPF takes the following form:

$$\begin{aligned} P_2 &= R_2R_{21}R_1, \\ P_3 &= R_3R_1(R_{31}+Q_{31}R_{32}R_2R_{21}), \\ P_4 &= R_4R_1(R_{41}+Q_{41}R_{42}R_2R_{21} \\ &+Q_{42}R_{43}R_3(R_{31}+Q_{31}R_{32}R_2R_{21})), \end{aligned} \quad (7)$$

An obtained value of P_4 is the probability of message delivery through channels A B. For k-th node the following form recurrently calculates from formula (4).

To calculate the numerical value of the probability of message delivery through the channel a, required to set the vector probabilities of availability nodes and links of communication network A — vector $R_A = \{r_1^A, r_2^A, r_3^A, r_4^A, \ldots, r_k^A\}$ and vector $S_A = \{r_{ij}^A, \ldots, r_{43}^A, r_{42}^A, r_{41}^A, r_{32}^A, r_{31}^A, r_{21}^A\}, 2 \leqslant i \leqslant k, 1 \leqslant j \leqslant k-1$. For channel technology b state of nodes and links B determine by the values of vectors:$R_B = \{r_1^B, r_2^B, r_3^B, \ldots, r_k^B\}$ and $S_B = \{p_{ij}^B, \ldots, p_{43}^B, p_{42}^B, p_{41}^B, p_{32}^B, p_{31}^B, p_{21}^B\}$. Probability of availability nodes and links set in the range: $0 \leqslant r_i \leqslant 1$ and $0 \leqslant r_{ij} \leqslant 1$ channels for both networks technologies.

After all substitution to R_A, S_A and R_B, S_B, we obtain the values P_k^A and P_k^B.

The higher of the two values P_k for networks A and B determine the channel a or b node r_i, on which message will be transmitted.

5. Conclusion

The search for the optimal route of transferring data during limited time interval is a key problem of implementing a cloud–oriented mobile mesh–network [2, 3]. Logical–probabilistic method of building evaluation of messaging routes allows to shorten the time needed to calculate the probability of delivering a message over a given channel and define the best route of delivering message at any moment. The power of such an approach is in ability to find an analytic solution in common case with incomplete input data about network elements condition. This paper contains an example with calculation of the probability of message delivery for the network of fully connected acyclic graph. The example demonstrates the solution for the problem of searching a new route in the case of data transfer session interruption when transmission time is greater than network lifetime, which is important when transmitting large amount of data by high–level protocols.

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