Fermionic Dark Matter Plus Baryons in Dwarfs Galaxies

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A generalization of the Jeans equations in the context of galactic dynamics is developed
for a multi-component self-gravitating system composed of dark matter particles and stars.
In addition to the luminous profile, an underlying fermionic phase–space density for the
dark component is assumed. Under the ansatz of isotropy, spherical symmetry and constant
dispersion velocities, this approach is applied to typical well resolved nucleated dwarf
galaxies, to obtain novel dark matter density profiles showing central mass concentrations at
pc distance–scales. Narrow constraints on the mass of the dark matter candidate of \( m \sim 1 \) keV are obtained.

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1. Introduction

In the realm of galactic dynamics, baryonic and Dark Matter (DM) components are usually
treated in the literature in terms of the Jeans equations (see e.g., [1] for a full development
of the theory and next section for a brief introduction). When dealing with dwarf galaxies
it is usually assumed that the underlying gravitational potential \( \Phi(r) \) in halo regions is
dominated by the DM component. This ansatz together with assumptions of time-independent
systems, spherical symmetry with no angular momentum dependence and constant line-of-
sight velocity dispersions \( \sigma_{\text{los}} \) (LOSVD), allows to break the Jeans degeneracy appearing in
anisotropic systems (see e.g. [1]). In this case, it is possible to fully solve the Jeans equations
to express the DM density profile in terms of the observables: \( \sigma_{\text{los}} \) and \( \Sigma(R) \), the last
being the surface brightness (see e.g. [2] for a theoretical approach on this matter, and [3] for
a phenomenological approach).

The main motivation of this work is based on observations of the well resolved nucleated
dwarf galaxies as observed and studied in [4], together with the novel theoretical approach
introduced in [5] and [6] to model the distribution of DM in normal galaxies. This approach relies
on a self-gravitating system of thermal and semi-degenerate keV fermions, whose equilibrium
configurations predicts a dark compact object (below parsec scales) plus an extended DM halo
providing flat rotation curves in agreement with observations. Thus, the objective of this work
is to provide a theoretical background based on the Jeans equations to deal in a self-consistent
way with baryonic and DM components from the center up to the halo of well resolved dwarf
galaxies. In this new picture, the assumption of considering an overall gravitational potential
dominated by the DM component has to be relaxed to properly account for the gravitational
effect of the baryons towards the center.

The nucleated regions (at pc scales) arising in the majority of dwarfs galaxies is a non-well
understood issue (see e.g., [4] and references therein). Therefore, the final objective of this
more general approach, even under the ideal and simplifying hypothesis adopted, is to give more
light on this matter by providing an underlying
fermionic phase–space distribution for the dark component which naturally condenses through the center due to quantum pressure.

2. Generalized formalism for a system of DM plus baryons

The specific Jeans equation under the general assumptions of time-independent systems in spherical symmetry with no angular momentum dependence reads (see e.g. [1] chapter 4):

\[
\frac{d}{dr}(\nu(r)v^2_r) = -\nu(r)\frac{d}{dr}\Phi(r). \tag{1}
\]

Here \(\Phi(r)\) is the gravitational potential, \(v^2_r\) is the mean square radial velocity and \(\nu(r) \equiv g/h^3 \int d^3v f(r,v^2)\) is the probability density to find a given component of the system at \(r\); in the last \(f(r,v^2)\) is a corresponding phase–space distribution function, \(g\) is a particle state degeneracy, and \(h\) is the Planck constant.

The equation (1) is a hydrostatic equilibrium-like equation, and differs only in that \(\nu(r)\) represents a probability density instead of a mass density, and that the mean particle velocity replaces the fluid velocity; being therefore \(\nu(r)v^2_r\) the pressure-like term.

The objective of this section is to generalize the hydrostatic equilibrium-like equation (1) to a more general case of a multiple-component system of point masses in dynamical equilibrium. For definiteness, we will consider a self-gravitating system composed by \(N_1\) identical collisionless dark matter particles and \(N_2\) identical collisionless stars, neglecting any possible interaction (other than gravitational) between both kinds of matter. Therefore, within this more general effective treatment, we can write the analogous of Eq. (1) but in terms of the mass density and pressure terms as follows:

\[
\frac{d}{dr}P_T(r) = -\rho_T(r)\frac{d}{dr}\Phi_T(r), \tag{2}
\]

where \(P_T(r)\) and \(\rho_T(r)\) are the total pressure and total mass density of the multi-component system composed by DM particles and stars (index \(T\) is for total hereafter in text).

The following step is to write Eq. (2) in terms of each pressure and density components. This is, by assuming the following decompositions \(P_T(r) = P_L(r) + P_{DM}(r)\) and \(\rho_T(r) = \rho_L(r) + \rho_{DM}(r)\), Eq. (2) reads

\[
\frac{dP_L(r)}{dr} + \frac{dP_{DM}(r)}{dr} + (\rho_L + \rho_{DM}(r)) \frac{d\Phi_T(r)}{dr} = 0. \tag{3}
\]

Because we are here assuming a possible linear independence between the gravitational effects of each component, plus the non-interacting (other than gravity) nature between the two matter components, we can write (3) as a system of two ordinary differential equations as follows,

\[
\frac{d}{dr}(j(r)\sigma^2_r) = -j(r)\frac{d}{dr}\Phi_T(r), \tag{4}
\]

\[
\frac{d}{dr}P_{DM}(r) = -\rho_{DM}(r)\frac{d}{dr}\Phi_T(r). \tag{5}
\]

We have expressed the luminous pressure \((P_L(r) \equiv j(r)\sigma^2_r)\) in terms of the three-dimensional luminosity density \(j(r)\) and the (luminous) radial dispersion velocity \(\sigma_r\). The luminosity density also appears in the right side of the equation for consistency, when considering we also adopt here a constant mass-to-light ratio \(\Upsilon = \text{const}\), defined as \(\rho_L(r) = \Upsilon j(r)\). Notice that under the symmetries adopted here, the (luminous) dispersion velocity \(\sigma_r\) coincides with both, the corresponding mean square radial velocity and also with the observable \(\sigma_{los}\) (see e.g. [1]).

The above system of equations has to be considered together with the Poisson equation,

\[
\nabla^2\Phi_T(r) = 4\pi G \rho_T(r). \tag{6}
\]

With the aim of obtaining a unique equation which contain the information of the system (4–5), we divide both equations to eliminate the gravitational gradient, and separate each matter
component functions at each side of the new equation, to obtain,
\[
\frac{1}{\rho_{DM}(r)} \frac{d}{dr} P_{DM}(r) = 1 \frac{j}{j(r)} \frac{d}{dr} j(r) \sigma^2_r. \tag{7}
\]

The equation (7) will be considered from now as a Jean master equation, containing the information of both kinds of matter which self-gravitates in an unique system.

We turn now to deal with the equation of state of the dark matter model.

Here we will limit to deal with the parametric equation of state of a non-relativistic self-gravitating Fermi gas, as this physical regime is more than sufficient when dealing with normal galaxies, and in particular as in this work, dwarf galaxies. Thus we have (the spin degeneracy has been taken \( g = 2 \)),
\[
\rho_{DM}(r) = \frac{m^4}{\pi^2 \hbar^3} \int_0^\infty v^2 f_{DM}(r, v^2) dv, \tag{8}
\]
\[
P_{DM}(r) = \frac{1}{3} \frac{m^4}{\pi^2 \hbar^3} \int_0^\infty v^4 f_{DM}(r, v^2) dv, \tag{9}
\]
where \( f_{DM}(r, v^2) \) is given by
\[
f_{DM}(r, v^2) = \frac{1}{\exp[(mv^2)/(2kT) - \theta(r)] + 1}. \tag{10}
\]

where \( m \) is the fermion mass, \( T \) is (some constant) temperature of the isothermal dark matter component, \( \theta(r) = \mu(r)/kT \) is the degeneracy parameter defined in terms of the gravitationally coupled chemical potential \( \mu(r) \), and \( k \) is the Boltzmann constant. The infinite integrals in (8–9) can be expressed in terms of the Polylogarithmic special functions \( \text{Li}_s(z) \) of order \( s \) and argument \( z \). Considering that \( \text{Li}_s(-z) = -\Gamma(s)^{-1} \int_0^\infty dt t^{s-1} [\exp(t)/z + 1] \), with \( t = mv^2/(2kT), z = \exp(\theta(r)), \) and \( s = 3/2 \) or \( s = 5/2 \) in correspondence with (8) or (9) respectively. If the following property for the derivative of the Polylogarithm \( d[\text{Li}_s(z(r))] / dr = z'(r)/z(r) \text{Li}_{s-1}(z(r)) \) is used, we directly have for the left side in Eq. (7)
\[
1 \rho_{DM}(r) \frac{d}{dr} P_{DM}(r) = kT \frac{d}{m} \frac{d}{dr} \theta_j(r). \tag{11}
\]

The equation (11) will be considered from now as a dark matter master equation, containing only information about the equation of state of the dark matter component.

Thus, we now combine the two master equations (7) and (11) in one unique equation given by,
\[
1 \frac{j}{j(r)} \frac{d}{dr} (j(r) \sigma^2_r) = kT \frac{d}{m} \theta_j(r). \tag{12}
\]

It is important to notice that the equation (12) is an ordinary linear differential equation in \( \theta_j(r) \), this last being interpreted as the degeneracy parameter affected by the gravitational effect of the baryonic distribution \( j(r) \). Notice that on the left side of Eq. (12) we have the observables, and on the right side we have the parameters of the dark matter component \((T, \theta_j, m)\), with \( T \) the DM temperature which must be found to fully solve the equations.

Once the solution for the degeneracy parameter \( \theta_j(r) \) is obtained from the observables \( \sigma \equiv \sigma_r \) and \( j(r) \), this must be replaced in the Polylogarithm variant of the equation (8) to yield the following important expression for the dark matter density function,
\[
\rho_{DM}(r) = -\frac{m^{5/2}(kT)^{3/2}}{\sqrt{2\pi^{3/2}h^3}} \text{Li}_{3/2}(-\exp(\theta_j(r))). \tag{13}
\]

3. Application to nucleated dwarf galaxies

The dwarf galaxies are an excellent astrophysical laboratory to study distribution and nature of dark matter particles because they belong to the most dark matter-dominated objects in the Universe as demonstrated in [7]. Recently, in [4] a big sample of about 70 dwarf galaxies in the Coma cluster (of distance \( D = 100 \) Mpc) were analyzed from high-resolution spectroscopic and photometric data, evidencing a nucleated luminosity profile.
through the center in the majority of the cases. It is believed that this central light excess is an imprint of the formation history of these galaxies, but there is no closed explanation of the causes and processes which leads to this new structure at pc distance-scales or below. The application of the generalized approach here introduced pretends to give more light to this important issue.

The nucleated surface brightness profiles observed in dwarf galaxies [4], are typically modeled by a Sérsic+Gaussian model of the form (see also Fig. 1)

\[
\frac{\Sigma(R)}{(L/pc^2)} = \Sigma_0 e^{-0.5(R/R_c)^2} + \Sigma_e e^{-b(R/R_e-1)},
\]

where \(\Sigma_0\) is the central observed value of the surface brightness and \(\Sigma_e\) is the effective surface brightness, while \(R_c\) and \(R_e\) are the central core scale radius and the effective radius respectively. It is important to notice that the Sérsic index \(n\) in (14) has been taken equal to unity as it is representative of the majority of the sample considered in [4]. The value of \(b\) depends on \(n\) (see e.g. [8]), and in the cases analyzed here (i.e. \(n = 1\)) it is \(b \approx 1.66\). The three dimensional luminosity density profile \(j(r)\) is obtained through the Abel de-projection formula to yield the following analytic expression,

\[
\frac{j(r)}{(L/pc^3)} = \frac{1.25}{\pi} \frac{\Sigma_0}{R_e} e^{-0.5(r/R_c)^2} + \frac{7.92}{\pi} \frac{\Sigma_e}{R_e} K_0(1.6r/R_e)
\]

where \(K_0(x)\) is the modified Bessel function of the second kind and of order 0. We adopt typical values of luminosity and scale-radii in dwarfs as shown in [4]: \(\Sigma_0 = 560 \text{ L}_\odot/\text{pc}^2\), \(\Sigma_e = 40 \text{ L}_\odot/\text{pc}^2\),

\[\text{FIG. 1. A double-component Sérsic-Gaussian model fit to a typical observed surface brightness } \Sigma(R) \text{ in } (L_\odot/pc^2) \text{ as considered in [4].}\]

\[\text{FIG. 2. Two different dimensionless dark matter density profiles in correspondence with the free parameters } \theta^0_j = -0.2 \text{ and } \theta^0_j = -0.7 \text{ implying a dark matter dominance of } \sim 90\% \text{ and } \sim 50\% \text{ at } R_e \text{ respectively. These dark profiles are obtained from the dynamical multi-component approach here developed and are contrasted against the total mass density profile as obtained directly from the observables, nicely showing how light follows dark matter all along the configuration.}\]
$R_c = 25$ pc, $R_e = 850$ pc. The constant line-of-sight velocity dispersion adopted here is $\sigma_{\text{los}} = 9$ km/s, according to [4], thus implying a total (integrated) mass-to-light ratio of $\Upsilon = 1.6$ as obtained from [4] (see Fig. 12 of that paper).

The DM temperature $T$ needed to finally solve Eqtn. (12) is obtained by assuming DM predomination in the halo region, where clearly the Maxwellian regime in the Fermi-Dirac distribution function is reached (i.e. $\mu(r)/kT << -1$). Therefore we must have necessarily $T \approx m\sigma_{\text{DM}}^2/k$, where $\sigma_{\text{DM}}$ is the DM one-dimensional dispersion velocity. Now, $\sigma_{\text{DM}}$ can be obtained from the flat part of the DM rotation curve proper of this classical regime, where the following relation holds (see e.g. [1]) $v_{\text{circ}} = \sqrt{2\sigma_{\text{DM}}}$. With this we have the desired DM temperature as $T = m^2 v_{\text{circ}}^2/(2k)$. In what follows we adopt $v_{\text{circ}} = 13$ km/s, being a typical circular velocity in dwarf galaxies as shown in [3]; leading to $kT/m = v_{\text{circ}}^2/2$ (km/s)$^2$, most important in order to solve Eq. (12).

The function $\theta_j(r)$ is obtained by integration of the equation (12) between $r_0$ and $r$, with $\sigma_r^2 = 81$ (km/s)$^2$ and $kT/m = 84.5$ (km/s)$^2$, to yield $\theta_j(r) = 0.95 \ln [(j(r)/j(r_0)) + \theta_j(r_0)]$, with $j(r_0) = 4$ pc$^3$. Therefore we have necessarily the innermost resolved radius for a typical dwarf galaxy as studied in [4].

Once with the solution for $\theta_j(r)$, and by using Eq. (13) together with the total mass density $\rho_T(r) = \Upsilon_j(r)$, it is possible to obtain the ratio between them. This ratio is calculated in dimensionless units to obtain an expression only in terms of the free parameter $\theta_j^0$ (i.e. independently of the fermion mass). For this, Eq. (13) is normalized dividing by $\rho_{\text{DM}}^n = m^4 v_{\text{circ}}^3/(4\pi^{3/2}k^3)$, while $\rho_T^0(r)$ is normalized dividing by the central total mass density $\rho_0^0 \approx 20M_\odot/pc^3$. Therefore the new normalized formulas are

$$\rho_n^{DM}(r) = -Li_{3/2}[(j(r)/j_0)^{0.95}e^{-\theta_j^0}]; \quad (16)$$
$$\rho_n^T(r) = \Upsilon_j(r)/\rho_0^T. \quad (17)$$

In Fig. 2 we show the (normalized) total mass density profile typical of a nucleated dwarf galaxy in the Coma cluster $\rho_n^T(r)$ together with two different $\rho_n^{DM}(r)$ for two different values of $\theta_j^0$. The value of $\theta_j^0 = -0.4$ is selected assuming a dark matter dominance of $\sim 94\%$ at $R_e$ (and a dominance of $\sim 55\%$ at $r_0$), while $\theta_j^0 = -0.7$ corresponds for a dark matter dominance of $\sim 70\%$ at $R_e$ (with a $\sim 42\%$ at $r_0$). Values of $\theta_j^0 > 0$ are prohibited because otherwise the dark matter density would overcome the total mass density.

Once the precise DM dominance at the center of the configuration $r_0$ is known, the ‘ino’ mass $m$ can be obtained from the DM density equation $\rho_n^{DM}(r)$ together with the normalization factor $\rho_{\text{DM}}^n$. The calculations in the two cases here assumed $\theta_j = -0.4$ and $\theta_j = -0.7$ leads respectively, to rest fermion masses of $m = 1.15$ keV/$c^2$ and $m = 1.14$ keV/$c^2$; implying a small effect of few $10^2$ eV/$c^2$ due to the different dark matter halo dominance adopted. These mass values have to be seriously considered only as an order of magnitude due to many different simplifying assumptions adopted such as spherical symmetry and constant $\sigma_{\text{los}}$ and $\Upsilon$, being not necessarily the case in real dwarf galaxies.

4. Conclusions

In conclusion, from this two-component (DM plus stars) dynamic approach, and due to semi-degenerate nature of the dark matter phase–space adopted here, it is possible to better understand the so-called central light excess observed in the light profiles of many dwarf galaxies. This is, the fact that the dominating DM component condenses through the center due the (fermionic) quantum pressure, it generates a deepen in the gravitational potential well in which the baryonic component naturally falls in, generating as a response a nucleated behavior in the light profile we observe at pc distance-scales or below. The second, and most important outcome of this approach, is that once the dark matter dominance is known at the effective radius $R_e$, the ‘ino’ mass
value can be obtained from the equations, falling in the keV region.

Moreover, we want to emphasize the potential importance of this approach in views of future high-resolution observations through the center of nearby dwarfs which will reach sub-pc distance-scales; leading to a better understanding in the role of dark matter in connection with massive dark central objects generally interpreted as intermediate massive black holes.

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References